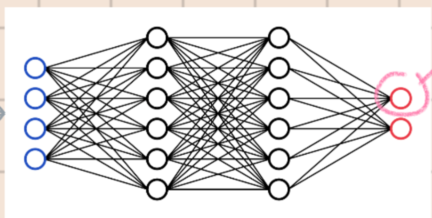
Introduction to Linear Algebra

Platforms like Google, ChatGPT (or any AI), or even Games — all have something in common, and that is they all use Linear Algebra in some way.

For example:

when you provide any type of data into AI (ex. chat GPT)

or any other type of data



neural network (ML Transformation)

this data is inserted into a neural network to go through the AI's algorithm

① (The process behind AI)

[2, -7, 8, ...]

vector

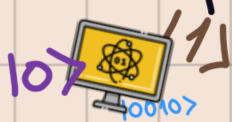
every neuron in the neural network is represented by a vector/matrix

②

Photos, videos, 3D characters, ... all these list under the sience of computer graphics which depends heavily on Linear Algebra.

③

States and representations in Quantum Computing also depends on linear Algebra.



④

Grayscale images are represented by / made of 2D matrices & RGB images also is represented by 3D matrices.

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Matrices (matrix)

↳ regtangular arrays.

each matrix have a dimension: n × m

/ an order: (rows × columns)

matrices're usually written in square brackets or parantheses.

$$\begin{pmatrix} a_{11} & \cdots & \cdots \\ a_{21} & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

ex.

$$\begin{array}{cc} & \\ \text{3 rows} & \begin{bmatrix} 2 & 4 & 0 \\ 0 & -3 & 6 \\ 0 & 0 & 8 \end{bmatrix} \\ & \text{3 columns} \end{array}$$

dimension / order

3×3

Types of matrices:

Row matrix:

↳ no. of rows = 1

↳ order = 1 × m

ex. $\begin{bmatrix} a & b & c & d \end{bmatrix}$ 1×4

Column matrix:

↳ no. of columns = 1

↳ order = n × 1

ex. $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ 3×1

Zero/Null matrix:

↳ a matrix that all its elements are zeros.

ex. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$

Symmetric matrix:

↳ must be:

1. $a_{ij} = a_{ji}$ 2. $A = A^T$
3. square matrix (n=m)

ex. $\begin{bmatrix} a & 1 & 2 \\ 1 & b & 3 \\ 2 & 3 & c \end{bmatrix}$

Note

matrix notation: $[a_{ij}]$, where i is the row & j is the column. The diagonal of a matrix are the elements where i = j.

Diagonal matrix:

↳ must be:

1. square matrix
2. $a_{ij} = 0$, for $i \neq j$

↳ (i.e. all elements in the matrix are zeros except the diagonal elements.)

ex. $\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$

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Identity matrix:

↳ a diagonal matrix but with all the diagonal elements being ones. (I_n) no. of rows/columns

ex. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

Remember:

we have 2 ways of describing physical quantities

vector — has — magnitude, direction

scalar — only has — magnitude

- the vectors are the matrices & scalars are numbers.

↳ when a scalar is multiplied by a vector/matrix, the multiplication is distributed to all the elements of the matrix.

[illegible]

ex. If r is a scalar.

$$(r) \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} r.a & r.b \\ r.c & r.d \end{bmatrix}$$

So, If A & B are matrices. Then:

- $-A = (-1).A$
- $A - B = A + (-1)B$

Adding & subtracting:

- just add/subtract each element to/from the one corresponding to it. $[a_{ij} \pm b_{ij}]$. But notice, the matrices you are subtracting/adding <u>must</u> be the SAME ORDER/dimension/size.

for example:

$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}^{2 \times 3}$ ✓, $B = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 4 & 5 \end{bmatrix}^{2 \times 3}$ ✓

$$A - 2B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 2 \\ 12 & 8 & 10 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -9 & -3 & -3 \end{bmatrix}$$

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Properties of matrices:

(A, B & C are matrices of same size, r & s are scalars)

- $A+B=B+A$
- $(A+B)+C=A+(B+C)$
- $A+0=A$

  null/zero matrix
- $r(A+B)=rA+rB$
- $(r+s)A=rA+sA$
- $r(sA)=(rs)A$
- $(A+B)^T=A^T+B^T$
- $(AB)^T=B^TA^T$
- $(AB)^{-1}=B^{-1}A^{-1}$
- $tr(rA)=r.tr(A)$
- $tr(A\pm B)=tr(A)\pm tr(B)$
- $tr(AB)=tr(BA)$
- $tr(B^{-1}AB)=tr(A)$
- $AB\neq BA$!!

Note
Transpose

the transpose of a matrix is when you switch rows with columns & viceversa.

$$a_{ij} \rightleftharpoons a_{ji}$$

↳ for example:

$$A=\begin{bmatrix}2 & 4\\6 & 7\end{bmatrix} \rightarrow A^T=\begin{bmatrix}2 & 6\\4 & 7\end{bmatrix}$$

Trace

↳ summation of diagonal also called trace or tr.

ex.

$$tr\left(\begin{bmatrix}2 & 3 & 5\\4 & 1 & 1\\0 & 0 & 7\end{bmatrix}\right)=2+1+7=10$$

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Multiplication of matrices:

↳ to multiply 2 matrices, the no. of columns in the 1st one has to be equal to the no. of rows in the 2nd.

i.e. $\underset{n \times m}{A} \quad \underset{m \times p}{B}$

has to be equal

dimension of AB

for each $\square_{ij}$ multiply the row (i) of the 1st to the (j) column of the 2nd.

ex. $A = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} (a_{11}b_{11} + a_{12}b_{21}) & (a_{11}b_{12} + a_{12}b_{22}) \\ (a_{21}b_{11} + a_{22}b_{21}) & (a_{21}b_{12} + a_{22}b_{22}) \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 24 \\ 15 & 22 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times2 + 2\times3 & 1\times5 + 2\times4 \\ 3\times2 + 4\times3 & 3\times5 + 4\times4 \end{bmatrix} = \begin{bmatrix} 8 & 13 \\ 18 & 31 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times1 + 2\times3 & 1\times2 + 2\times4 \\ 3\times1 + 4\times3 & 3\times2 + 4\times4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

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